International Journal of Theoretical Physics, Vol. 35, No. 8, 1996

Supersymmetric Anyon Model Coupled to the Electromagnetic Field

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Received September 15, 1995

We construct a supersymmetric gauge model describing the electromagnetic interaction of anyons. This is done by means of the supersymmetric generalization of the $U(1) \times U(1)$ gauge theory. The model contains the statistical $U(1)$ gauge field endowed with a Chern–Simons mass term and the electromagnetic field, both with the corresponding superpartners, coupled to matter fields. This constrained system is analyzed from the Hamiltonian point of view and the canonical quantization is found. The path-integral method is used to develop the perturbative formalism. We define suitable propagators and vertices and give the diagrammatics and the Feynman rules.

1. INTRODUCTION

Recently, by starting from the general classical $U(1) \times U(1)$ nonrelativistic gauge theory which describes the electromagnetic interaction of anyons (Cortes *et al.*, 1994), we have studied the model from the quantum point of view (Foussats *et al.*, 1995b). Using the path-integral method, we found the Feynman rules and the diagrammatics of this model.

As is well known, in 2 + 1 dimensions, when time reversal and parity invariance are violated, it is possible to have anyons. Anyons are important not only from the theoretical point of view, but also phenomenologically. There are several models and different theoretical approaches for describing anyonic excitations (Berezin and Marinov, 1977; Goldin *et al.*, 1980, 1981; Wilczek, 1982; Wilczek and Zee, 1983; Laughlin, 1983; Chern *et al.*, 1991; Wu and Zee, 1984; Bowick *et al.*, 1986; Dzyaloshinskii *et al.*, 1988; Polyakov,

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0020-7748/96/0800-1679$09.50/0

1988; Plyushchay, 1992; see Wilczek, 1991, for review). The most fruitful model is constructed by coupling minimally a bosonic or fermionic system to a $U(1)$ statistical gauge field. In this type of model, the dynamics is governed by the Chern–Simons (CS) action (Hagen, 1984, 1985a,b; Arovas *et al.*, 1985; see Jackiw, 1990, for review).

Approaches are also available including the electromagnetic interaction of anyons (Kogan, 1991; Kogan and Semenoff, 1982; Stern, 1991; Cabo *et al.*, 1992; Cortes *et al.*, 1992; Chou *et al.*, 1993; Chaichian *et al.*, 1993). The electromagnetic interactions of anyons in the framework of quantum field theory can be formulated as a $U(1) \times U(1)$ gauge theory coupled to a matter (bosonic or fermionic) field through a conserved current (Cortes *et al.*, 1994).

For the case of nonrelativistic field theory, Cortes *et al.* (1994) showed how it is possible to obtain the value $\mu_{st} = 1/(2m)$ of the statistical magnetic moment required when the one-particle sector is considered. Of course, the $U(1) \times U(1)$ gauge model also reproduces the value $\mu_{em} = (e/m)s$ for the electromagnetic magnetic moment. So, to reproduce the results from quantum mechanical formulations, a simple and elegant way is to couple an anticommuting (or commuting) matter field to two $U(1)$ gauge fields. One of these is the electromagnetic field and the other the statistical field introduced through the CS action.

When an anyon system coupled to an electromagnetic field is considered in the framework of the relativistic quantum field theory formalism, additional problems associated with the nonlocalized character of the statistical current appear. These questions have been discussed by Cortes *et al.* (1994) (see also Frölich and Marchetti, 1988).

On the other hand, Hlousek and Spector (1990) give an interesting supersymmetric formulation of pure anyon theories. Starting from the standard formalism of pure anyon theories in terms of the $U(1)$ statistical CS field, the minimal supersymmetric model with fractional spin and statistics is constructed. Of course, the first result is that supersymmetry connects fields of spin S with fields of spin $S + 1/2$. When the particle content and the interactions of the model are explored, an anyon–anyon interaction required by supersymmetry naturally appears. So, the main conclusion is that in this type of model the interaction among anyons is a direct requirement of supersymmetry. More precisely, in a supersymmetric theory, anyon species must interact in order to preserve supersymmetry. This fact can be seen from a general point of view by defining a minimal coupling among a suitable conserved current superfield and a gauge spinor superfield.

Hlousek and Spector (1990) also carried out a complete study for the case of topological solitons having fractional spin and statistics. This is done by constructing the supersymmetric generalization of the Hopf term for the supersymmetric $O(3)$ nonlinear sigma model. The results reflect the importance of considering supersymmetry when fractional spin and statistics are present.

Following this line, in the present paper we will focus on formulating the classical and quantum supersymmetric generalized version of the $U(1) \times U(1)$ gauge theory coupled to matter fields previously developed in Cortes *et al.* (1994) and Foussats *et al.* (1995b). In the constructive procedure of this supersymmetric model we will apply the techniques used in classical and quantum CS theories in 2 + 1 dimensions coupled to different kinds of matter fields (Deser *et al.*, 1982a,b, 1988; Dunne *et al.*, 1989; Jackiw and Templeton, 1981; Matsuyama, 1990a,b; Lin and Ni, 1990; Avdeev *et al.*, 1992; Odintsov, 1992).

The paper is organized as follows. In Section 2, the definitions and quantities we need to construct the classical supersymmetric action in a superspace are introduced. In Section 3, we analyze the constraint structure of the model and we find the extended Hamiltonian of the coupled constrained supersymmetric system. Next, by following the Dirac algorithm, we carry out the canonical quantization in a straightforward way. In Section 4, we construct the perturbative formalism by using the path-integral method.

2. PRELIMINARIES AND CLASSICAL SUPERSYMMETRIC ACTION

In pure anyonic models (see, for instance, Hagen, 1984, 1985a,b; Arovas *et al.*, 1985; Jackiw, 1990) one can consider a charged spin-1/2 Dirac field ψ or a complex spin-0 field φ coupled in the standard way to a $U(1)$ statistical gauge field A_μ. The gauge field A_μ must be included as a CS mass term, i.e.,

$$\mathcal{L}_f = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + \frac{1}{2\sigma}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \tag{2.1}$$

or

$$\mathcal{L}_b = D_\mu\varphi^* D^\mu\varphi - m^2\varphi^*\varphi + \frac{1}{2\sigma}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \tag{2.2}$$

where the gauge-covariant derivative is $D_\mu = \partial_\mu - igA_\mu$, and σ is the statistical parameter. The convention used in $\epsilon^{012} = \epsilon^{12} = 1$, and the Minkowski metric $g_{\mu\nu}$ is $g_{\mu\nu} = \text{diag}(1, -1, -1)$. The Dirac γ-matrices are $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, and $\gamma^2 = i\sigma^2$ (the σ's are the Pauli matrices).

As is well known, the Lagrangian (2.1) describes the theory of particles with spin $S = (e^2/4\pi)\sigma + 1/2$ obeying the fractional statistics $(e^2/2\pi)\sigma + 1$, and the Lagrangian (2.2) describes the theory of particles with spin $S = (e^2/4\pi)\sigma$ obeying the fractional statistics $(e^2/2\pi)\sigma$, in both cases for arbitrary values of σ, while for $\sigma = \pm 2\pi$ the Bose–Fermi transmutation occurs and the value of the spin is well defined.

We remark that in equations (2.1) and (2.2) a Maxwell term $-\frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A)$ may or may not be present. The CS term breaks both parity and time-reversal invariance and makes it possible to turn these into anyonic Lagrangians. That is, (2.1) and (2.2) describe field theories and particles with fractional statistics, due to the presence of the CS term.

On the other hand, an anyon system interacting with the electromagnetic field can be considered (Cortes *et al.*, 1994). From a general point of view and taking into account only general requirements of gauge invariance, it is possible to construct a $U(1) \times U(1)$ gauge theory. In this type of model a matter field couples through a conserved current and the anyon dynamics interacting with the electromagnetic field involves two $U(1)$ gauge fields. Having in mind this model, in Foussats *et al.* (1995b) we carried out the quantization and also developed the perturbation theory.

Now, our purpose is to construct the supersymmetric generalization of the $U(1) \times U(1)$ gauge theory. To do this, we first introduce some definitions in the superspace of coordinates (x^μ, θ), where θ is a two-component Majorana spinor (see, for instance, Grisaru, 1982; Gates *et al.*, 1983).

Looking at the Lagrangian density of the $U(1) \times U(1)$ gauge theory (Cortes *et al.*, 1994; Foussats *et al.*, 1995b), it can be seen that to maintain supersymmetry, the supersymmetric completion procedure must be applied. The supersymmetrization of the matter fields is easily obtained by simply adding the free-field Lagrangians (2.1) and (2.2). Next, we must find the supersymmetric partner of the CS term. Besides this, we must consider the supersymmetric generalization of the following electromagnetic Lagrangian [see equation (2.9) of Foussats *et al.* (1995b)]:

$$\mathscr{L}_{\text{em}} = -\frac{1}{4} F_{\mu\nu}(B)F^{\mu\nu}(B) - \frac{1}{8\pi}\frac{e}{m} F_{\mu\nu}(A)F^{\mu\nu}(B) \tag{2.3}$$

where B_μ is the electromagnetic field.

Consequently, to describe the supersymmetry involving the matter fields, we will use complex scalar superfields $\Phi(x^\mu, \theta)$ and $\Phi^*(x^\mu, \theta)$. A complex scalar superfield has components

$$\Phi(x^\mu, \theta) = \varphi(x^\mu) + \theta^\alpha\psi_\alpha(x^\mu) - \theta^2 F(x^\mu) \tag{2.4}$$

where $\varphi(x^\mu)$ is a complex scalar field, $\psi_\alpha(x^\mu)$ is a Majorana spinor field, and $F(x^\mu)$ is also an auxiliary scalar field.

To describe the supersymmetry involving the two gauge fields $U(1)$, we will use spinor superfields. In the spinor superfields the gauge connections

A_μ or B_μ and their corresponding superpartners are embedded. In the general definition of spinor superfields, a couple (scalar boson–fermion) of purely gauge objects can be included. The simplest case is called Wess–Zumino gauge, in which the purely gauge objects are taken equal to zero. For simplicity, we will work in the Wess–Zumino gauge and therefore write a spinor superfield $V_\alpha(x^\mu, \theta)$ in components:

$$V_\alpha(x^\mu, \theta) = -i(\gamma^\mu\theta)_\alpha A_\mu - 2\theta^2\lambda_\alpha \tag{2.5}$$

where only the gauge field A_μ and its superpartner λ_α are included. Of course, another spinor superfield for the gauge connection B_μ and the corresponding superpartner χ_α must be also defined.

Therefore, to generalize the $U(1) \times U(1)$ gauge theory in the Wess–Zumino gauge, a gauge-invariant superpartner λ (gaugino) for the gauge boson A_μ, and another gauge-invariant superpartner χ (photino) for the gauge field B_μ, are necessary.

As usual, the supercovariant derivative D_α (SCD) in the superspace is defined by $D_\alpha = \partial/\partial\theta^\alpha + i(\gamma^\mu\theta)_\alpha\partial_\mu$. A gauge transformation in superspace is given by $V_\alpha \to V_\alpha + D_\alpha\Lambda$ and $\Phi \to \exp(ie\Lambda)\,\Phi$, where $\Lambda(x^\mu, \theta)$ is any real-valued function of superspace.

The definition of the gauge-covariant supercovariant derivative (GCSD) used in pure supersymmetric anyon theories is given by the equation $\nabla_\alpha = D_\alpha - ieV_\alpha(A, \lambda)$ where $V_\alpha(A, \lambda)$ plays the role of superconnection (Hlousek and Spector, 1990). When the electromagnetic interaction is present, we define the $U(1) \times U(1)$ GCSD containing a double superconnection as follows:

$$\nabla_\alpha = D_\alpha - ie[V_\alpha(A, \lambda) + V_\alpha(B, \chi)] \tag{2.6}$$

Another necessary object in the supersymmetric generalization is the field strength spinor superfield given by

$$W_\alpha = \tfrac{1}{2}D^\beta D_\alpha V_\beta \tag{2.7}$$

The field strength superfield (2.7) makes possible the supersymmetric generalization of two kinds of terms: the CS term by coupling the gauge connection superfield to the field strength spinor superfield, as well as the two terms in equation (2.3) by coupling the superfield strength with the superfield strength.

As shown in Hlousek and Spector (1990) for pure anyon theories, the minimal supersymmetric Lagrangian obtained by direct generalization of the nonsupersymmetric case is obtained from the action:

$$\mathcal{S}^{\text{an}}_{\text{minimal}} = \frac{1}{2}\int d^3x\, d^2\theta(\overline{\nabla}^\alpha\Phi^*)(\nabla_\alpha\Phi) - \int d^3x\, d^2\theta\, m\Phi^*\Phi + \frac{1}{4\sigma}\int d^3x\, d^2\theta\, \overline{V}^\alpha(A, \lambda)W_\alpha(A, \lambda) - \frac{1}{4}\int d^3x\, d^2\theta\, \overline{W}^\alpha(A, \lambda)W_\alpha(A, \lambda) \tag{2.8}$$

where the massive term was added for convenience; but setting $m = 0$ is possible and the conclusions are unchanged.

Writing the minimal action (2.8) in components in the Wess–Zumino gauge, and integrating out the Grassmann variables θ, we obtain the minimal Lagrangian density in the form

$$\mathcal{L}^{\text{an}}_{\text{minimal}} = -\frac{1}{4}F_{\mu\nu}(A)F^{\mu\nu}(A) + \frac{i}{2}\overline{\lambda}\gamma^\mu\partial_\mu\lambda + D_\mu\varphi^* D^\mu\varphi - m^2\varphi^*\varphi + i\overline{\psi}(\gamma^\mu D_\mu - m)\psi + ie(\overline{\psi}\lambda\varphi - \overline{\lambda}\psi\varphi^*) - \frac{1}{2\sigma}\overline{\lambda}\lambda + \frac{1}{2\sigma}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho \tag{2.9}$$

It is easy to see that the introduction of the electromagnetic interaction naturally yields the following minimal action:

$$\mathcal{S}_{\text{minimal}} = \frac{1}{2}\int d^3x\, d^2\theta\, (\overline{\nabla}^\alpha\Phi^*)(\nabla_\alpha\Phi) - \int d^3x\, d^2\theta\, m\Phi^*\Phi + \frac{1}{4\sigma}\int d^3x\, d^2\theta\, \overline{V}^\alpha(A, \lambda)W_\alpha(A, \lambda) - \frac{1}{4}\int d^3x\, d^2\theta\, \overline{W}^\alpha(B, \chi)W_\alpha(B, \chi) - \frac{1}{16\pi}\frac{e}{m}\int d^3x\, d^2\theta\, [\overline{W}^\alpha(B, \chi)W_\alpha(A, \lambda) + \overline{W}^\alpha(A, \lambda)W_\alpha(B, \chi)] \tag{2.10}$$

The use of the superfields Φ, V_α, and W_α and the GCSD ∇_α, all defined in the superspace, guarantees the supersymmetry of the action (2.10). The gauge invariance $U(1) \times U(1)$ of the model is ensured by construction.

Once the integration on θ coordinates is performed in equation (2.10), the total Lagrangian density is given by

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{an}} + \mathcal{L}_{\text{em}} \tag{2.11}$$

The Lagrangian density in components in the Wess–Zumino gauge reads

$$\begin{aligned}\mathcal{L}_{\text{an}} = {} & D_\mu\varphi^* D^\mu\varphi - m^2\varphi^*\varphi + i\bar{\psi}(\gamma^\mu D_\mu - m)\psi \\ & + ie(\bar{\psi}\lambda\varphi - \bar{\lambda}\psi\varphi^*) + ie(\bar{\psi}\chi\varphi - \bar{\chi}\psi\varphi^*) \\ & + \frac{1}{2\sigma}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{1}{2\sigma}\bar{\lambda}\lambda \end{aligned} \tag{2.12}$$

$$\begin{aligned}\mathcal{L}_{\text{em}} = {} & -\frac{1}{4}F_{\mu\nu}(B)F^{\mu\nu}(B) - \frac{e}{8\pi m}F_{\mu\nu}(A)F^{\mu\nu}(B) \\ & + \frac{i}{2}\bar{\chi}\gamma^\mu\partial_\mu\chi + \frac{ie}{8\pi m}(\bar{\chi}\gamma^\mu\partial_\mu\lambda + \bar{\lambda}\gamma^\mu\partial_\mu\chi)\end{aligned} \tag{2.13}$$

where now $D_\mu = \partial_\mu - ieA_\mu - ieB_\mu$.

The Lagrangian (2.11) describes the interacting theory of a field φ of spin $S = (e^2/4\pi)\sigma$ and a field ψ of spin $S = (e^2/4\pi)\sigma + 1/2$ (and their conjugates), both interacting at the same time with the electromagnetic field.

The particular form of the couplings among the matter fields φ and ψ is also a property of the supersymmetric anyon Lagrangian (2.12). Moreover, when $e \to 0$ the interaction terms go away while supersymmetry remains. When the supersymmetry is also eliminated the model is reduced to a pure (fermionic or bosonic) anyon system maintaining fractional spin and statistics (see, for instance, Polyakov, 1988; Kogan, 1991; Kogan and Semenoff, 1992). From (2.12) it can be seen how the supersymmetry naturally produces an anyon–anyon interaction through the coupling to the two photino λ and χ superpartners, respectively, of the two $U(1)$ gauge fields A_μ and B_μ. The photino mass term $(1/2\sigma)\bar{\lambda}\lambda$ superpartner for the CS term is also present, serving as a gauge-invariant mass term for the gauge field A_μ. Therefore, all these facts are direct consequences of the supersymmetric requirements of the model.

Moreover, we notice that when the electromagnetic interaction is present, the statistical gauge field A_μ as well as the two photinos turn into dynamical fields. So, none of the photinos can be integrated out as occurs in pure anyon theories with λ when the model has only the CS term for A_μ (Hlousek and Spector, 1990).

Finally, we briefly comment that the presence of the term $(e/8\pi m)F_{\mu\nu}(A)F^{\mu\nu}(B)$ in (2.13) in the nonsupersymmetric case is derived from general arguments of $U(1) \times U(1)$ gauge invariance. More precisely, when the dynamics of anyons interacting with the electromagnetic field is written in terms of conserved current, an additional contribution $J_\mu(B)$ to the statistical current is needed. The requirement to obtain the correct value for the electromagnetic moment uniquely determines the piece of current $J_\mu(B)$ depending on the electromagnetic field B_μ, and therefore the coupling of both $U(1)$ gauge fields remains univocally determined by

$$J^\mu(B)A_\mu = \frac{e}{8\pi m} F_{\mu\nu}(A)F^{\mu\nu}(B) \tag{2.14}$$

The same procedure in terms of conserved current can be carried out in the supersymmetric case. A spinor superfield conserved current $\mathcal{J}^\alpha$ ($D_\alpha \mathcal{J}^\alpha = 0$) minimally coupled to the gauge spinor superfield V_α can be defined. So, writing the action $\frac{1}{2}\int d^3x\, d^2\theta\, \mathcal{J}^\alpha V_\alpha$ in components, in the Wess–Zumino gauge, besides (2.14) the following superpartner term is obtained:

$$\frac{ie}{8\pi m} (\bar{\chi}\gamma^\mu \partial_\mu \lambda + \bar{\lambda}\gamma^\mu \partial_\mu \chi) \tag{2.15}$$

We notice that these two terms are quadratic respectively in the gauge fields and in the photino fields, but both terms are linear in each one of the fields. As will be seen, these terms play an important role in the definition of propagators in the quantum formalism.

In the next section we will analyze the constraint structure with the aim of finding the extended Hamiltonian of the model.

3. CONSTRAINT STRUCTURE HAMILTONIAN AND GAUGE-FIXING CONDITIONS

Now we briefly study this coupled system in the framework of the Dirac formalism for constrained Hamiltonian systems. It is interesting to analyze the constraint structure, the gauge-fixing conditions, and all the arguments needed to develop the perturbative method starting from the path-integral formalism.

The phase space is constructed by starting from the total Lagrangian (2.11). The momenta canonically conjugate to the independent field variables are given by

$$P_A^0 = 0 \tag{3.1a}$$

$$P_A^i = \frac{e}{4\pi m} F^{i0}(B) + \frac{1}{2\sigma} \epsilon^{ij} A_j \tag{3.1b}$$

$$P_B^0 = 0 \tag{3.1c}$$

$$P_B^i = F^{i0}(B) + \frac{e}{4\pi m} F^{i0}(A) \tag{3.1d}$$

$$P_\varphi = \partial_0 \varphi - ie(A_0 + B_0)\varphi \tag{3.1e}$$

$$P^*_\varphi = \partial_0\varphi^* + ie(A_0 + B_0)\varphi^* \tag{3.1f}$$

$$\overline{\Pi}_\psi = -i\overline{\psi}\gamma^0 \tag{3.1g}$$

$$\Pi_\psi = 0 \tag{3.1h}$$

$$\overline{\Pi}_\lambda = -\frac{ie}{8\pi m}\overline{\chi}\gamma^0 \tag{3.1i}$$

$$\Pi_\lambda = 0 \tag{3.1j}$$

$$\overline{\Pi}_\chi = -\frac{i}{2}\left(\overline{\chi} + \frac{e}{4\pi m}\overline{\lambda}\right)\gamma^0 \tag{3.1k}$$

$$\Pi_\chi = 0 \tag{3.1l}$$

where the Latin indices take the values $i, j = 1, 2$.

The Poisson brackets between pairs of canonical conjugate variables are as usual (see, for instance, Sundermeyer, 1982) and so they are not written here.

Looking at equations (3.1), we see that (3.1a) and (3.1c) are the primary bosonic constraints and (3.1g)–(3.1l) are the primary fermionic ones. From these constraints we are able to construct the total classical Hamiltonian $H_T = \int d^2x\, \mathcal{H}_T$ generator of time evolution, where the Hamiltonian density $\mathcal{H}_T$ is given by

$$\mathcal{H}_T = \mathcal{H}_{\text{can}} + b_A P_A^0 + b_B P_B^0 + \bar{f}_\psi\Pi_\psi + (\overline{\Pi}_\psi + i\overline{\psi}\gamma^0)f_\psi + \bar{f}_\lambda\Pi_\lambda + \left(\overline{\Pi}_\lambda + \frac{ie}{8\pi m}\overline{\chi}\gamma^0\right)f_\lambda + \bar{f}_\chi\Pi_\chi + \left[\overline{\Pi}_\chi + \frac{i}{2}\left(\overline{\chi} + \frac{e}{4\pi m}\overline{\lambda}\right)\gamma^0\right]f_\chi \tag{3.2}$$

Here b_A, b_B and $\bar{f}$, f are, respectively, bosonic and fermionic Lagrange multipliers.

As usual, the functional $\mathcal{H}_{\text{can}}$ is defined by

$$\mathcal{H}_{\text{can}} = \dot{A}_\mu P_A^\mu + \dot{B}_\mu P_B^\mu + \dot{\varphi}P_\varphi + \dot{\varphi}^*P_\varphi^* + \dot{\overline{\psi}}\Pi_\lambda + \dot{\psi}\overline{\Pi}_\psi + \dot{\overline{\lambda}}\Pi_\lambda + \dot{\lambda}\overline{\Pi}_\lambda + \dot{\overline{\chi}}\Pi_\chi + \dot{\chi}\overline{\Pi}_\chi - \mathcal{L}$$

which, after using (3.1b) and (3.1d)–(3.1f), we can write as

$$\mathcal{H}_{\text{can}} = \left(\frac{4\pi m}{e}\right)\left(\frac{2\pi m}{e}P_A^i - P_B^i\right)P_{iA} + \frac{1}{2\sigma}\left(\frac{4\pi m}{e}\right)\left(\frac{4\pi m}{e}P_A^j - P_B^j\right)\epsilon_{ij}A^i + \frac{1}{8\sigma^2}\left(\frac{4\pi m}{e}\right)^2 A_jA^j + \partial_iA^0P_A^i + \partial_iB^0P_B^i - \frac{1}{2\sigma}\epsilon^{ij}\partial_iA_jA^0$$

$$
\begin{aligned}
&+ \frac{e}{8\pi m} F_{ij}(A)F^{ij}(B) \\
&+ \frac{1}{4} F_{ij}(B)F^{ij}(B) + P_\varphi^* P_\varphi + ie(A^0 + B^0)(\varphi P_\varphi^* - \varphi^* P_\varphi) \\
&- e\overline{\psi}(A^0 + B^0)\gamma^0\psi - i\overline{\psi}(\gamma^i D_i - m)\psi - (D^i\varphi)^*(D_i\varphi) + m^2\varphi^*\varphi \\
&- \frac{i}{2}\overline{\chi}\gamma^i\partial_i\chi \\
&- \frac{ie}{8\pi m}(\overline{\chi}\gamma^i\partial_i\lambda + \overline{\lambda}\gamma^i\partial_i\chi) + \frac{1}{2\sigma}\overline{\lambda}\lambda - ie(\overline{\psi}\lambda\varphi - \overline{\lambda}\psi\varphi^*) \qquad (3.3) \\
&- ie(\overline{\psi}\chi\varphi - \overline{\chi}\psi\varphi^*)
\end{aligned}
$$

Analyzing the set of primary constraints from equation (3.1), it is possible to show that there are other two bosonic secondary constraints of second class. Therefore, besides the six second-class fermionic constraints, the model has initially four bosonic constraints, two of which are first class and two second class. By direct computation we can find linear combinations of constraints giving rise to two other first-class bosonic constraints. Thus, the final set of constraints is given as follows:

(i) The four bosonic first-class constraints are

$$\Sigma_1 = P_A^0 \approx 0 \qquad (3.4a)$$

$$\Sigma_2 = P_B^0 \approx 0 \qquad (3.4b)$$

$$\Sigma_3 = e\left(\partial_i P_A^i + \frac{1}{2\sigma}\epsilon^{ij}\partial_i A_j\right) - e\partial_i P_B^i \approx 0 \qquad (3.4c)$$

$$\Sigma_4 = -\frac{i}{e}\partial_i P_B^i + \varphi^* P_\varphi - \varphi P_\varphi^* + \overline{\psi}\Pi_\psi + \overline{\Pi}_\psi\psi \approx 0 \qquad (3.4d)$$

(ii) We call the six fermionic second-class constraints Ω_a ($a = 1, 2, \ldots, 6$), and they are given in equations (3.1g)–(3.1l).

When the Dirac algorithm is continued and the consistency condition is imposed on the fermionic second-class constraints, the corresponding Lagrange multipliers $\bar{f}$ and f in equation (3.2) remain univocally determined.

Consequently, in the framework of the canonical quantization, the quantum Hamiltonian functional of the constrained system under consideration is written as

$$H_T^* = \int d^2x\,(\mathcal{H}_{can} + a\Sigma_1 + b\Sigma_2 + c\Sigma_3 + d\Sigma_4) \qquad (3.5)$$

where a, b, c, and d are undetermined parameters, and the four associated first-class constraints correspond to all the gauge symmetries of the model.

The procedure can be continued by constructing the Dirac brackets from the Poisson ones. To complete the canonical quantization the second-class constraints must be taken as equations strongly equal to zero. Finally, the Dirac brackets are replaced into the equal-time (anti) commutators according to the usual rule (Dirac, 1964).

In the system under consideration, four first-class constraints Σ_i remain, so, in order to restrict the system on the true phase space, subsidiary conditions must be imposed. These conditions, or gauge-fixing conditions $F_i \approx 0$ (one for each first-class constraint), must be compatible with the equation of motion. In addition, they must satisfy the following requirements: for all the first-class constraints $\Sigma_i \approx 0$, $\det[f_i, \Sigma_j]_D \neq 0$ and $[f_i, f_j]_{PB} = 0$. This means that the conditions $F_i \approx 0$ and $\Sigma_i \approx 0$ are all independent and really restrict the phase space. The above requirements do not determine uniquely the gauge-fixing conditions F_i, consequently, as an example, we can choose the following simplest expressions:

$$F_1 = \partial_i A^i \approx 0 \tag{3.6a}$$

$$F_2 = \partial_i B^i \approx 0 \tag{3.6b}$$

$$F_3 = \nabla^2\left(B_0 + \frac{e}{4\pi m} A_0\right) + e\bar{\psi}\gamma^0\psi + ie(\varphi^* P_\varphi - P^*_\varphi \varphi) \approx 0 \tag{3.6c}$$

$$F_4 = \nabla^2 B_0 + \frac{4\pi m}{e}\left[\frac{1}{\sigma}\epsilon^{ij}\partial_i A_j + e\bar{\psi}\gamma^0\psi + ie(\varphi^* P_\varphi - P^*_\varphi \varphi)\right\} \approx 0 \tag{3.6d}$$

When the quantization procedure is implemented by using the path-integral method, the above gauge-fixing conditions will be used explicitly. They play an important role in determining the part of the gauge-fixed action $\mathcal{S}_{\text{fix}}$ in the total effective quantum action given by $\mathcal{S}_q = \mathcal{S}_{\text{class}} + \mathcal{S}_{\text{fix}}$. In general $\mathcal{S}_{\text{fix}}$ results quadratic in the gauge-fixing conditions F_i, i.e., $\mathcal{S}_{\text{fix}} = \frac{1}{2}F_i c^{ij} F_j$.

4. QUANTIZATION AND PERTURBATIVE METHOD

In this section, we construct a perturbative method by defining a proper diagrammatic, propagators and vertices, in the framework of the path-integral formalism. As the coupled system we are analyzing has first- and second-class constraints, the simplest way is to proceed according to the Faddeev–Senjanovič formalism (Faddeev, 1970; Senjanovic, 1976). So, we assume that the partition function for the $U(1) \times U(1)$ supersymmetric gauge model can be written as follows:

$$Z = \int \prod \mathcal{D}(\mathrm{BF})\, \mathcal{D}(\mathrm{BM})\, \mathcal{D}(\overline{\mathrm{FF}})\, \mathcal{D}(\mathrm{FM})\, \mathcal{D}(\mathrm{FF})\, \mathcal{D}(\overline{\mathrm{FM}})\, \delta(\Sigma_i)$$
$$\times\, \delta(F_i) \det[\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, F_1, F_2, F_3, F_4]_{\mathrm{D}}\, \delta(\overline{\Omega}_a)\, \delta(\Omega_a) \det[\overline{\Omega}_a, \Omega_b]$$
$$\times \exp i\left[\int d^3x\, (\dot{A}_\mu P_A^\mu + \dot{B}_\mu P_B^\mu + \dot{\varphi} P_\varphi + \dot{\varphi}^* P_\varphi^* + \dot{\bar{\psi}}\Pi_\psi + \dot{\psi}\overline{\Pi}_\psi\right.$$
$$\left. + \dot{\bar{\lambda}}\Pi_\lambda + \dot{\lambda}\overline{\Pi}_\lambda + \dot{\bar{\chi}}\Pi_\chi + \dot{\chi}\overline{\Pi}_\chi) - H_T\right] \tag{4.1}$$

where the Hamiltonian density H_T was already defined in (3.2). We have written (BF) = boson fields, (BM) = boson momenta, (FF) = fermion fields, and (FM) = fermion momenta.

In equation (4.1), the matrix whose elements are $[\Sigma_i, f_j]_D$ is written as follows:

$$[\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, F_1, F_2, F_3, F_4]_{\mathrm{D}} = \begin{pmatrix} 0 & 0 & 0 & -\frac{e}{4\pi m}\nabla^2 \\ 0 & 0 & -\frac{e}{4\pi m}\nabla^2 & -\nabla^2 \\ e\nabla^2 & -e\nabla^2 & 0 & 0 \\ 0 & -\frac{i}{e}\nabla^2 & 0 & 0 \end{pmatrix} \delta(x-y) \tag{4.2}$$

The determinant of the matrix (4.2) is

$$\det[\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4, f_1, f_2, f_3, f_4]_{\mathrm{D}} = i\left(\frac{e}{4\pi m}\right)^2 (\nabla^2)^4 \delta(x-y) \tag{4.3}$$

as it does not depend on the field variables, so it is included in the path-integral normalization factor.

Just the same occurs with the other determinant appearing in (4.1), constructed from the second-class constraints.

Using in equation (4.1) the delta functions $\delta(\Sigma_1)$, $\delta(\Sigma_2)$, $\delta(\overline{\Omega}_a)$, and $\delta(\Omega_a)$, we immediately perform the path integral over the fields P_A^0, P_B^0, $(\overline{\mathrm{FM}})$ and (FM). Consequently, after the integration is carried out, the Hamiltonian H_T appearing in the exponential of the action of equation (3.1) becomes H_{can}.

On the other hand, the two gauge-fixing conditions (3.6c) and (3.6d) can formally be solved for A_0 and B_0, namely

$$B_0(x) \approx p_2 = -\frac{m}{e}\int d^2y\, \frac{[(1/\sigma)\epsilon^{ij}\partial_i A_j + e\bar{\psi}\gamma^0\psi + ie(\varphi^* P_\varphi - P_\varphi^*\varphi)]}{|x-y|} \tag{4.4a}$$

$$A_0(x) \approx -\frac{4\pi m}{e} B_0 + p_1 = -\frac{m}{e} B_0 - m\int d^2y\, \frac{[\bar{\psi}\gamma^0\psi + i(\varphi^* P_\varphi - P_\varphi^*\varphi)]}{|x-y|} \tag{4.4b}$$

In equation (4.1) the delta functions can be written $\delta(F_3) = \delta(A_0 + (4\pi m/e)B_0 - p_1)$ and $\delta(F_4) = \delta(B_0 - p_2)$ and so the path integral on A_0 and B_0 also can be performed.

Consequently, the Hamiltonian H_{can} remaining in the exponential of equation (3.1) can be partitioned as follows:

$$\begin{aligned} H_{\text{can}} &= H_0 - A_0\left(\frac{1}{e}\Sigma_1 + i\Sigma_2\right) - ieB_0\Sigma_2 \\ &= H_0 + \left(\frac{4\pi m}{e^2} p_2 - \frac{1}{e} p_1\right)\Sigma_1 - i\left(p_1 - \frac{4\pi m}{e} p_2 + ep_2\right)\Sigma_2 \end{aligned} \tag{4.5}$$

Now, the integral representation $\delta(\Sigma_\alpha) = \int \mathcal{D}\Lambda_\alpha \exp(i\int d^3x\, \Lambda_\alpha\Sigma_\alpha)$ ($\alpha = 1, 2$) can be introduced. Therefore, taking into account the arbitrariness of the multipliers Λ_α and following the usual steps, it is possible to rescale the corresponding integration variables in such a way as to recover the original H_{can} (Sundermeyer, 1982).

Performing the Gaussian integrals over the remaining momentum variables, we find the final form of the partition function (4.1):

$$\begin{aligned} Z = \int & \mathcal{D}A_\mu\, \mathcal{D}B_\mu\, \mathcal{D}\varphi\, \mathcal{D}\varphi^*\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, \mathcal{D}\bar{\lambda}\, \mathcal{D}\lambda\, \mathcal{D}\bar{\chi}\, \mathcal{D}\chi \\ & \times \delta(\partial_i A^i)\delta(\partial_i B^i) \exp i\left[\int d^3x\, \mathcal{L}_{\text{eff}}\right] \end{aligned} \tag{4.6}$$

where $\mathcal{L}_{\text{eff}}$ is the original Lagrangian written in (2.11).

Finally, using the Faddeev–Popov trick to go over a general covariant gauge $\partial_\mu A^\mu = c_A(x)$ and $\partial_\mu B^\mu = c_B(x)$, we find the final form of the partition function (4.6):

$$Z = \int \mathcal{D}A_\mu\, \mathcal{D}B_\mu\, \mathcal{D}\varphi\, \mathcal{D}\varphi^*\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, \mathcal{D}\bar{\lambda}\, \mathcal{D}\lambda\, \mathcal{D}\bar{\chi}\, \mathcal{D}\chi \exp i\left[\int d^3x\, \mathcal{L}^*\right] \tag{4.7}$$

The functional $\mathcal{L}^*$ is given by

$$\mathcal{L}^* = \mathcal{L}_{\text{eff}} + \mathcal{L}_{\text{fix}} \tag{4.8}$$

where

$$\mathcal{L}_{\text{fix}} = \frac{\lambda_A}{2}(\partial^\mu A_\mu)^2 + \frac{\lambda_B}{2}(\partial^\mu B_\mu)^2 \tag{4.9}$$

Looking at equation (4.7), we can see that a fruitful form for the partition function was obtained. The quantum problem was written in terms of a path integral over all the independent dynamical fields. Subsequently, the problem can be treated using the diagrammatic technique in the framework of Feynman path-integral perturbation theory. In principle it is straightforward to go from the path integral (4.7) to the Feynman rules, propagators, and vertices ('t Hooft and Velman, 1973).

In this supersymmetric gauge model, the key to the interaction between both $U(1)$ gauge fields, or their corresponding superpartners, is respectively given by the term (2.14) or (2.15) appearing in the Lagrangian (2.13). As noted above, these interacting kinetic quadratic terms must contribute to propagators. Such terms, being quadratic in the fields but linear in each of them, takes part of the corresponding propagators in an unusual way.

So, due to the presence of these terms in the functional $\mathcal{L}^*$, the only possibility is to construct from the two gauge fields a unique bosonic mixed propagator associated with an extended bosonic quantity. Analogously, a mixed fermionic propagator also can be defined.

We define the auxiliary quantity $X_\Lambda \equiv (A_\mu, B_\nu)$, where the compound index $\Lambda \equiv (\mu, \nu)$ takes six values. Correspondingly, we introduce the auxiliary fermionic quantity $\Xi \equiv (\lambda, \chi)$.

Then, after we write the action in terms of these quantities, we recognize the quadratic part of the Lagrangian $\mathcal{L}^*$ as representing the propagators and the remaining pieces as representing the vertices. Consequently, the Lagrangian density (4.8) defines the effective action of an anyon system coupled to the electromagnetic field and it can be partitioned as follows:

$$\begin{aligned} S^* = {} & S^*(X_\Lambda) + S^*(\Xi) + S^*(\bar{\psi}, \psi) + S^*(\varphi^*, \varphi) \\ & + S^*_{\text{int}}(X_\Lambda, \Xi, \varphi^*, \varphi, \bar{\psi}, \psi) \end{aligned} \tag{4.10}$$

We have denoted

$$S^*(X_\Lambda) = \int d^3x \left[\frac{1}{2} X_\Lambda (D^{-1})^{\Lambda\Sigma} X_\Sigma\right] \tag{4.11a}$$

$$S^*(\Xi) = \int d^3x \, [\bar{\Xi} K^{-1} \Xi] \tag{4.11b}$$

$$S^*(\overline{\psi}, \psi) = \int d^3x\, [\overline{\psi} G^{-1} \psi] \tag{4.11c}$$

$$S^*(\varphi^*, \varphi) = \int d^3x\, [\varphi^* P^{-1} \varphi] \tag{4.11d}$$

$$\begin{aligned} S^*_{\text{int}}(X_\Lambda, \Xi, \varphi^*, \varphi, \overline{\psi}, \psi) &= \int d^3x\, [e^2 \varphi^* (X_\Sigma V^{\Sigma\Lambda} X_\Lambda) \varphi] \\ &+ \int d^3x\, ie[\overline{\psi} I \Xi \varphi - \overline{\Xi} I \psi \varphi^*] \\ &+ \int d^3x\, [e \overline{\psi} \Gamma^\Sigma \psi X_\Sigma] \\ &+ \int d^3x\, [2ie\varphi^* X_\Sigma \partial^\Sigma \varphi] \end{aligned} \tag{4.11e}$$

In equation (4.11a) the 6×6 matrix (D^{-1}) is the inverse of the propagator associated to the auxiliary field X_Λ, and it is Hermitian and nondegenerate. So the propagator $D_{\Lambda\Sigma}(k)$ in the momentum space can be evaluated as

$$D_{\Lambda\Sigma}(k) = \begin{pmatrix} M_{\mu\nu} & L_{\mu\nu} \\ L_{\mu\nu} & N_{\mu\nu} \end{pmatrix} \tag{4.12}$$

The quantities $M_{\mu\nu}$, $N_{\mu\nu}$, and $L_{\mu\nu}$ are given by

$$\begin{aligned} M_{\mu\nu} &= \left(\frac{4\pi m}{e}\right)^2 \alpha(k^2) g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \left[\left(\frac{4\pi m}{e}\right)^2 \alpha(k^2) + \frac{1}{\lambda_A} (\alpha(k^2) - \beta(k^2))\right] \\ &+ i\left(\frac{4\pi m}{e}\right)^2 \gamma(k^2) \epsilon_{\mu\nu\rho} \frac{k^\rho}{k^2} \end{aligned} \tag{4.13a}$$

$$\begin{aligned} N_{\mu\nu} &= \beta(k^2) g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \left[\beta(k^2) + \frac{1}{\lambda_B} (\alpha(k^2) - \beta(k^2))\right] \\ &+ i\gamma(k^2) \epsilon_{\mu\nu\rho} \frac{k^\rho}{k^2} \end{aligned} \tag{4.13b}$$

$$L_{\mu\nu} = -\frac{4\pi m}{e} \alpha(k^2) \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + i\frac{1}{\sigma}\left(\frac{4\pi m}{e}\right)^2 \epsilon_{\mu\nu\rho} \frac{k^\rho}{k^2}\right] \tag{4.13c}$$

and the functions $\alpha(k^2)$, $\beta(k^2)$, and $\gamma(k^2)$ are

$$\begin{aligned} \alpha(k^2) &= \left[k^2 - \frac{1}{\sigma^2}\left(\frac{4\pi m}{e}\right)^4\right]^{-1}, \qquad \beta(k^2) = \frac{1}{\sigma^2 k^2}\left(\frac{4\pi m}{e}\right)^4 \alpha(k^2) \\ \gamma(k^2) &= \frac{1}{\sigma}\left(\frac{4\pi m}{e}\right)^2 \alpha(k^2) \end{aligned} \tag{4.14}$$

As expected (Foussats *et al.*, 1995b), the form of the bosonic propagator (4.12) is the same as we previously obtained for the $U(1) \times U(1)$ nonsupersymmetric model.

In equation (4.11b) K^{-1} is the inverse of the propagator of the auxiliary fermionic field Ξ. The corresponding propagator $K(p)$ in the momentum space can be evaluated:

$$K(q) = \frac{1/(4\sigma) + (e/8\pi m)^2\gamma\cdot q}{(1/(4\sigma))^2 - (e/8\pi m)^4 q^2}\begin{pmatrix} -1/2 & e/8\pi m \\ e/8\pi m & -(1/2\sigma)\gamma\cdot q/q^2 \end{pmatrix} \tag{4.15}$$

Finally, in equations (4.11c) and (4.11d) P^{-1} and G^{-1} are the inverses of the propagators associated to the anyonic matter fields. In the momentum space, these propagators are given respectively by

$$G(p) = \frac{i(\gamma\cdot p - m)}{p^2 + m^2} \tag{4.16}$$

$$P(l) = \frac{1}{l^2 - m^2} \tag{4.17}$$

When the electromagnetic interaction is withdrawn, the pure anyonic model (Hlousek and Spector, 1990) is obtained. Of course, in such a case the path-integral method we have developed reduces to the usual one, giving rise to the well-known propagators.

Equation (4.11e) is the part of the action which accounts for the vertices of the model. There is a four-leg derivative vertex which is described by defining the 6×6 matrix $V^{\Sigma\Lambda}$,

$$V^{\Sigma\Lambda} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \tag{4.18}$$

The other vertices have three legs, one of which is derivative. Moreover, in equation (4.11e), we have formally written $I = (1, 1)$, $\Gamma^\Sigma = (\gamma^\mu, \gamma^\nu)$, and $\partial^\Sigma = (\partial^\mu, \partial^\nu)$.

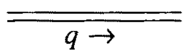
We can now write the Feynman rule propagators and vertices.

(i) *Propagators.* We associate with the propagator $D_{\Sigma\Lambda}$ of the bosonic field X_Σ a wavy line connecting two generic points,

X_Σ  $X_\Lambda \equiv D_{\Sigma\Lambda}(k)$

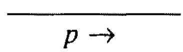
$k \rightarrow$

We associate with the propagator $K(q)$ of the fermionic field Ξ, the superpartner of X_Σ, a double line,



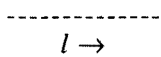
$\equiv K(q)$

We associate with the usual propagators of the fermionic and the bosonic matter fields a continuous and a dashed line, respectively:



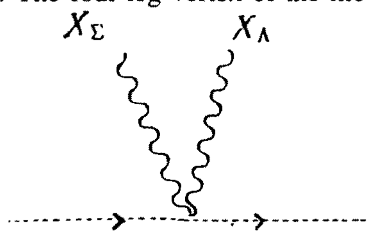
$\equiv G(p)$

and



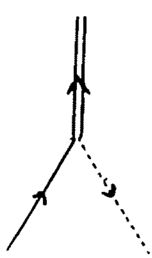
$\equiv P(l)$

(ii) *Vertices.* The four-leg vertex of the model is

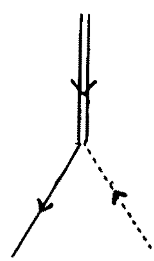


$\equiv e^2\, V^{\Sigma\Lambda}$

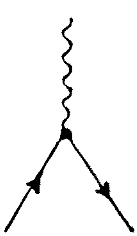
and the three-leg vertices are



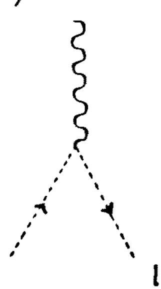
$\equiv iel,$



$\equiv -\, iel$



$\equiv e\Gamma^\Sigma,$



$\equiv 2iel^\Sigma$

Moreover, as usual, we have to take into account a minus sign for every closed fermion loop and another minus sign for diagrams related to the exchange of two fermion lines, internal or external. A combinatorial factor correcting for double counting in the case that identical particles occur also must be taken into account.

At this stage, we could analyze this supersymmetric gauge model describing the interaction of the pair of supersymmetric fields (X_Σ, Ξ) with the pair of supersymmetric matter fields (ψ, φ) in the framework of the perturbative theory.

We do not treat here the problem of regularization and renormalization of this model. However, by looking at the expressions of the propagators and taking into account the above Feynman rules, complete information about the perturbative behavior could be obtained. At least the one-loop structure can be easily studied by analyzing the superficial degree of divergence of the corresponding diagrams. It can be seen that this gauge model belongs to the class of theories with only a finite number of divergent diagrams. So the regularization and renormalization problem is reduced to the problem of regularizing a superrenormalizable theory and it can be done by the usual methods (Alvarez Gaumé *et al.*, 1990).

Finally, we note that when higher derivative terms are added to the action, the ultraviolet behavior of some propagators can be improved. This fact was already studied in nonsupersymmetric gauge models (Alvarez Gaumé *et al.*, 1990; Foussats *et al.*, 1995a).

Also in the present supersymmetric case it is shown that by adding in the action (2.10) the term

$$\int d^3x\, d^2\theta\, (\overline{\nabla}^\alpha \overline{W}^\beta(A, \lambda))(\nabla_\alpha W_\beta(B, \chi)) \tag{4.19}$$

which preserves the gauge invariance of the model, the behavior of the propagators (4.12) and (4.17) at large momentum can be improved. The new bosonic propagator associated with the field X_Σ gains two powers of k with respect to the form (4.12). Similarly, the new fermionic propagator associated with the field Ξ gains two powers of q with respect to the form (4.17). So, the useful result of this trick is to render the model less divergent.

5. CONCLUSIONS

Starting from the $U(1) \times U(1)$ gauge model for anyons interacting with the electromagnetic field, we have shown how it is possible to construct the supersymmetric version of the model.

The supersymmetric anyon model was treated as a constrained Hamiltonian system and the canonical quantization was found. The first-class constraints associated with the $U(1) \times U(1)$ symmetries were found. A set of compatible gauge-fixing conditions allowed us to determine the gauge-fixed part of the effective quantum action.

Next, by going over to path-integral quantization method, we could write the partition function and so construct the Feynman rules and the diagrammatics by defining suitable mixed propagators and vertices. Due to the form of the bilinear interacting terms, this supersymmetric gauge model admits a unique bosonic propagator associated with the two $U(1)$ gauge fields. Similarly, a unique mixed fermionic propagator associated with the two fermionic superpartners must be defined. Therefore, this supersymmetric anyon model can be treated in the framework of the perturbation formalism.

By means of the propagators thus defined, all the diagrams are obtained by connecting vertices and sources as usual.

The coupled system has different vertices; one of these has four legs and the remaining ones have three legs. The vertex structure is a direct consequence of the coupling properties of the supersymmetric Lagrangian.

Furthermore, looking at the diagrammatics, it is possible to conclude that the model belongs to the class of superrenormalizable theories because it has a finite number of divergent diagrams. As briefly remarked but not shown, by using the perturbative formalism developed, all the information and prescriptions about the regularization and renormalization of the model can be given.

ACKNOWLEDGMENTS

The authors would like to thank the Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina, for financial support.

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